



Attempt the following questions.

**Question 1:** (10 marks)

- a) Identify (i) what *Euclid's algorithm* does for a pair of integers in which the first is smaller than the second and (ii) how often this can happen during the algorithm's execution. (02 marks)
- b) Sort the following functions according to their order of growth from the lowest to the highest. (04 marks)  
 $n^3, n^{\log 5}, 2^n, n \log n, 3^{\log n}, n!, n^n, \log n^8, \log^3 n$
- c) Solve the following recurrence. (04 marks)

$$M(n) = 2M(n-1) + 1, M(1) = 1$$

**Question 2:** (10 marks)

- a) Construct an example of the assignment problem whose optimal solution does not include the smallest element of its cost matrix. (02 marks)
- b) Design and analyze a brute-force algorithm for finding a substring in a given text that matches a specified pattern. (04 marks)
- c) Design and analyze a brute-force algorithm for computing the value of a given polynomial  $p(x)$  at a given point  $x_0$  and determine its worst-case efficiency class. (04 marks)

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

**Question 3:** (10 marks)

- a) Justify whether it is possible to implement *insertion sort* for sorting *linked lists* **efficiently**. (03 marks)
- b) Design and analyze an efficient algorithm for the *exponentiation problem* (finding  $a^n$ ). (03 marks)
- c) Design and analyze an efficient algorithm for selecting the smallest  $k$  elements in a given array of size  $n$ . (04 marks)

**Question 4:** (10 marks)

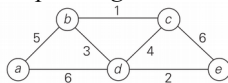
- a) Apply mergesort to the following array of characters. (02 marks)  
 $[E, X, A, M, P, L, E, S]$
- b) Design and analyze an efficient algorithm for estimating the height of a given binary tree. (03 marks)
- c) Given a set  $P$  of  $n > 1$  points in the Cartesian plane, design and analyze an efficient algorithm for finding the closest pair of points the given set. (05 marks)

**Question 5:** (10 marks)

- a) Compare between *Binary Trees*, *AVL Trees*, *Red-Black Trees*, *2-3 Trees*, and *B-Trees*. (02 marks)
- b) Design and analyze an efficient algorithm for checking element uniqueness in a given array. (04 marks)
- c) Design and analyze an efficient algorithm for computing the *least common multiple* (LCM) of two given integers  $m$  and  $n$ . (04 marks)

**Question 6:** (10 marks)

- a) Design and analyze a *dynamic programming algorithm* for the knapsack problem given  $n$  items of known weights  $w_1, \dots, w_n$  and values  $v_1, \dots, v_n$  and a knapsack of capacity  $W$ . (05 marks)
- b) Apply Kruskal's algorithm to find a *minimum spanning tree* of the shown graph. (02 marks)



- c) Design and analyze a *greedy algorithm* for the change-making problem given an amount  $n$  and coin denominations  $d_1 > d_2 > \dots > d_m$ . (03 marks)

Good Harvest  
 Dr. Islam ElShaarawy